On the Modeling and Optimization of Schottky Varactor Frequency Multipliers at Submillimeter Wavelengths

Jyrki T. Louhi and Antti V. Räisänen, *Fellow, IEEE*

Abstract—**Schottky varactor frequency multipliers are used to generate local oscillator power at millimeter and submillimeter wavelengths. The equivalent circuit of the Schottky varactor contains a junction capacitance, a junction conductance, a series resistance and a model for electron velocity saturation. At millimeter wavelengths the equivalent circuit is affected by the edge effects, which are due to the small-area circular anode. The correction factors due to the edge effect for the junction capacitance and for the series resistance are available in the literature. In this work the electron velocity saturation is modeled by limiting the velocity of the transition front between the depleted and undepleted layer. By using this model the maximum current of the diode is given by the actual area of the transition front between depleted and undepleted layers, and is therefore related to the capacitance correction factor. The new model has been tested by analyzing a two diode balanced doubler for 160 GHz presented earlier in the literature. The agreement between the theoretical results and the measurements is excellent. The new diode model is useful in optimization of varactors for high millimeter and submillimeter wave frequencies.**

I. INTRODUCTION

FREQUENCY MULTIPLIERS are used to generate the all-solid-state local oscillator power of heterodyne receivers at millimeter and submillimeter wavelengths [1]. These local oscillators are needed in many future scientific satellites (e.g., SWAS, Odin, FIRST, and SMIM). At millimeter and submillimeter wavelengths a Schottky varactor is the most commonly used multiplier device, although several novel varactors (SBV, QWD, BNN, bbBNN, HEMV) have been proposed [2].

Although the equivalent circuit of the Schottky varactor has been widely studied, varactor fabrication technology has been greatly improved, and many different kind of multiplier structures have been tried, the maximum output power of all-solid-state local oscillators remains low at submillimeter wavelengths. The aim of this work is to develop the equivalent circuit of the Schottky varactor in order to find a model, which is physically valid at millimeter and submillimeter wavelengths. The physical model is required, when THz-range Schottky varactor frequency multipliers are analyzed and optimized. The new model is especially necessary for the electron velocity saturation. In this work the electron velocity saturation has been related to the movement of the transition front between depleted and undepleted layers.

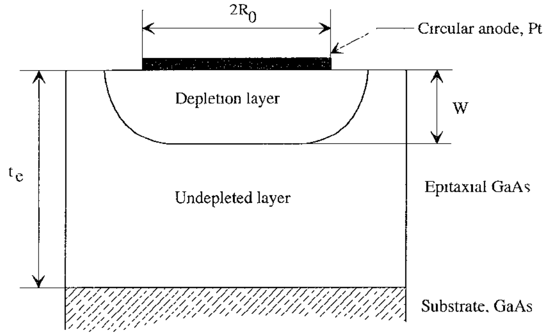

Fig. 1. Schematic of a Schottky varactor.

II. MODEL OF THE SCHOTTKY VARACTOR

The Schottky varactor consists of a circular metallic anode on top of an epitaxial GaAs layer as shown in Fig. 1. The radius of the anode is R_0 and the thickness of the epitaxial layer is t_e. At millimeter and submillimeter wavelengths the radius of the varactor anode is not large compared to the thickness of the epitaxial layer and to the width of the depletion layer. The potential of the epitaxial layer as well as the transition front between the depleted and undepleted layer are curved near the periphery of the circular anode as shown in Fig. 1. This edge effect should be taken into account in the equivalent circuit of the Schottky varactor, which includes a nonlinear junction capacitance C_j, a nonlinear junction conductance G_j and series resistance R_s as shown in Fig. 2(a). At millimeter and submillimeter wavelengths a model of the electron velocity saturation should also be included in the equivalent circuit of the Schottky varactor.

A. Junction Capacitance

The capacitance of the circular Schottky varactor can be found by calculating the net charge of the depletion layer from the numerically solved potential of the epitaxial layer [3], [4]. At millimeter wavelengths a first order correction term is normally included in the model of the junction capacitance as [3]–[5]

$$C_j = \frac{\epsilon A}{W}\left(1 + b\frac{W}{R_0}\right) \tag{1}$$

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The authors are with Helsinki University of Technology, Radio Laboratory, FIN-02150 Espoo, Finland.

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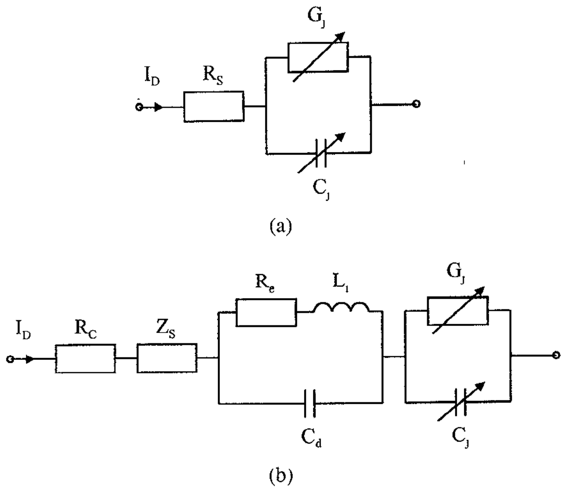

Fig. 2. Equivalent circuits of the Schottky varactor. (a) A simple circuit. (b) A circuit including the inertia inductance, the displacement capacitance, and the impedance of the substrate layer.

where ϵ is the dielectric constant of the semiconductor, A is the area of the anode, W is the width of the depletion layer and b is numerical constant 1.5. At submillimeter wavelengths an extra second order correction term should also be included in the model of the junction capacitance as [4]

$$C_j = \frac{\epsilon A}{W}\left(1 + b_1\frac{W}{R_0} + b_2\frac{W^2}{R_0^2}\right) = \frac{\epsilon A}{W}\gamma_C \tag{2}$$

where the numerical constants are $b_1 = 1.5$ and $b_2 = 0.3$, and γ_C is the net correction factor compared to the simple plate capacitance. The value of the net correction factor depends on the width of the depletion layer W. For a typical millimeter wave Schottky varactor the correction factor varies between values 1.0 ($W = 0$) and 1.5 ($W = t_e = R_0/3$) during a pump cycle.

B. Series Resistance

The series resistance of the partially depleted epitaxial layer can be found from the geometrical resistance of the undepleted layer. The exact solution of the resistance depends upon the width of the undepleted layer, which means that the series resistance is nonlinear [6]. However, if the nonlinearity of the series resistance is omitted, the efficiency of the multiplication is slightly decreased at low input power levels and remains almost the same at high power levels [7]. In computer simulations a constant series resistance is used in order to decrease the required time of the numerical analysis without decreasing the accuracy of the results too much. The resistance of the totally undepleted epitaxial layer with circular anode can be found analytically [8]

$$\begin{aligned} R_e &= \frac{\rho t_e}{A}\frac{1}{t_e}\int_0^\infty \frac{\tanh(zt_e)\sin(zR_0)J_1(zR_0)}{z^2}\,dz \\ &= \frac{\rho t_e}{A}\gamma_R, \end{aligned} \tag{3}$$

where ρ is the resistivity of the epitaxial layer and γ_R is the net correction factor due to the edge effects. For a varactor with a large anode compared to the thickness of the epitaxial layer ($R_0 \gg t_e$) the correction factor γ_R is 1. For a very small anode ($R_0 \ll t_e$) the series resistance is equal to the spreading resistance [9] and so the correction factor is given by

$$\gamma_R = \frac{\pi R_0}{4t_e}. \tag{4}$$

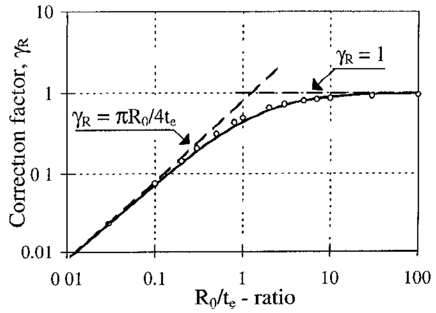

Fig. 3. The net correction factor of the series resistance. Solid line with approximative formula (5) and circles with exact formula (3).

For any R_0/t_e-ratio the net correction factor can be easily found from the approximative formula

$$\gamma_R \approx \left[1 + \frac{4t_e}{\pi R_0}\right]^{-1}. \tag{5}$$

For a typical millimeter wave varactor the radius of the anode is about three times the thickness of the epitaxial layer and so the net correction factor is 0.72 (see Fig. 3). The total series resistance of the varactor includes also the contact resistance R_c (about 1 Ω) and the series impedance of the substrate layer Z_s. At frequencies above 100 GHz the charge carrier inertia and dielectric displacement currents in the epitaxial layer should be included in the model of the series resistance by adding the inertia inductance L_i and displacement capacitance C_d to the model as [6]

$$L_i = R_e\frac{m^*\mu}{q}, \tag{6}$$

$$C_d = \frac{\epsilon\rho}{R_e} \tag{7}$$

where m^* is the effective mass of an electron, μ is the mobility of electron and q is the charge of an electron. The complete equivalent circuit of the submillimeter wave Schottky varactor is shown in Fig. 2(b).

C. Electron Velocity Saturation

At millimeter and submillimeter wavelengths the maximum output power of a Schottky varactor frequency multiplier is strongly affected by the electron velocity saturation, which takes place in GaAs at high electric fields. In an intrinsic case the maximum electron velocity is about 2.2×10^5 m/s at electric field of 3.2 kV/cm. Kollberg *et al.* have modeled this saturation of the electron velocity by a nonlinear resistance, which increases as the function of the electron current of the varactor [10]. This model is empirical, because the formula of the nonlinear resistance ($R_i = aR_S i^6$) is found by fitting the results of numerical analyses to the experimental results. East *et al.* have presented a more physical model, where the electron conduction current has been limited by the maximum current of the varactor [11]

$$I_m = AN_D q v_m \tag{8}$$

where N_D is the doping density, q is the charge of an electron and v_m is the maximum velocity of electrons. However this model is partially empirical, because the value of the maximum velocity (3.5×10^5 m/s) has been obtained by fitting the results of the numerical multiplier analyses to the experimental results.

In this work a new model has been derived for the electron velocity saturation by limiting the velocity of the transition front between the depleted and undepleted layer by the maximum velocity of the electron

$$\left|\frac{\partial W}{\partial t}\right| < v_m. \tag{9}$$

(This basic idea was already presented by Kollberg *et al.* [10], but was not further developed.) The width of the depletion layer is given by

$$W = \sqrt{\frac{2\epsilon(\phi_{bi} - V_j)}{qN_D}} \tag{10}$$

where ϕ_{bi} is built-in potential and V_j is the voltage over the Schottky junction. When the current required to pump the junction capacitance is

$$I_d = C_j \frac{\partial V_j}{\partial t} = C_j \frac{\partial V_j}{\partial W}\frac{\partial W}{\partial t} \tag{11}$$

the maximum current of the diode can be derived to be

$$I_m = AqN_D v_m \gamma_C. \tag{12}$$

The edge effect is taken into account by using the same correction factor γ_C, which is used for the junction capacitance. This means that the maximum current of the varactor depends on the actual area of the transition front. Therefore the maximum current of the varactor is always stronger than that obtained by using (8), because the area of the transition front is larger than the area of the varactor anode.

In a static situation the maximum electron velocity in GaAs is about 2.2×10^5 m/s at the electric field of 3.2 kV/cm, as mentioned before. However, during fast transients the electron velocity in GaAs can overshoot the steady-state value by several times [12], [13]. This velocity overshoot increases the maximum electron velocity at high frequencies. The actual effect of the velocity overshoot depends on many device physical, geometrical and operating properties. It can only be determined by thorough electron transport simulations. According to numerical simulations, the maximum velocity at millimeter wavelengths can be assumed to be about 2.9×10^5 m/s at electric field of 5 kV/m [14]. Without decreasing the accuracy of the multiplier simulations too much we have used a simple expression for the velocity versus field, which has a constant mobility at low fields and a constant maximum velocity at higher fields.

TABLE I
PARAMETERS OF THE MILLIMETER AND SUBMILLIMETER WAVE SCHOTTKY VARACTORS (UVa PARAMETERS FROM [10], [16])

Varactor type		UVa 6P4	UVa 2T2	Optimized	
Radius of the anode	R_0	3.2	1.4	1.4	μm
Thickness of the epitaxial layer	t_e	1.0	0.5	0.2	μm
Doping density	N_D	$3.5 \cdot 10^{16}$	$10 \cdot 10^{16}$	$10 \cdot 10^{16}$	cm^{-3}
Electron mobility	μ	0.61	0.55	0.55	m^2/Vs
Series resistance	R_s	10	12.0	~7	Ω
Junction capacitance at zero bias	C_{j0}	21	6.5	6.5	fF

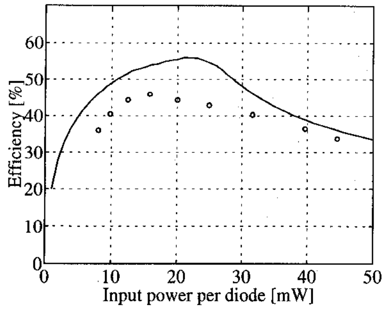

Fig. 4. Maximum theoretical efficiency of a 160-GHz doubler (solid line). Measured results, after correction for 0.5-dB input losses and 0.8-dB output losses, have also been plotted (circles).

III. MULTIPLIER PERFORMANCE

A. Doubler for 160 GHz

We have tested the new model by analyzing the two diode balanced doubler for 160 GHz constructed by Erickson [15]. The varactor used in the doubler is a whisker contacted UVa 6P4 Schottky varactor, whose parameters are shown in Table I. The doubler has been analyzed by using the multiple reflections technique, where the linear part of the circuit is solved in frequency domain and the nonlinear part is solved in time domain [17].

The results of our analyses are shown in Fig. 4. The efficiency has been calculated with optimum embedding impedances and with an optimum negative bias voltage. The results of our theoretical analyses agree well with the experimental results, when the estimated losses for the multiplier mount are taken into account (0.5-dB input losses and 0.8-dB output losses) [18]. The equality between the numerical and experimental result is very good at high power levels, when the effect of the electron velocity saturation is high. The disagreement between the numerical results and the experiments at low input power levels can be partly explained by the measured VSWR at the input waveguide (VSWR is about 1.9–2.7 at input power levels 20–10 mW per diode) [18].

According to the simulation results the maximum current during a pump cycle is about 66 mA, when the input power is 60 mW per diode and the embedding impedances are $Z_1 = (50 + j180)\Omega$, $Z_2 = (90 + j110)\Omega$. This means that the maximum current obtained with the one-dimensional model (about 52 mA) is exceeded by 25%. In other words, when the edge effects have been taken into account, the increased area of the transition front between the depleted and undepleted layer helps to pump the junction capacitance of the Schottky varactor more efficiently than can be assumed, if (8) is employed.

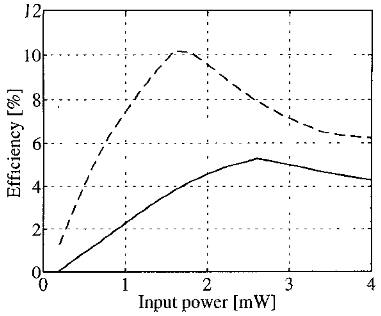

Fig. 5. Maximum theoretical efficiency of a 960-GHz tripler. Solid line: UVa 2T2; dashed line: optimized varactor.

B. Multiplier Chain for THz-Range

The new model for electron velocity saturation can be used to analyze all submillimeter wave Schottky varactor frequency multipliers (doubler, tripler, quadrupler, quintupler, etc.), because it is independent from the multiplication factor and from the type of the Schottky varactor. As an example we have analyzed the tripler for 960 GHz, because the multiplier chain utilizing a doubler for 160 GHz followed by a doubler for 320 GHz and a tripler for 960 GHz seems to be a good choice to generate the all-solid-state local oscillator for frequencies at near 1 THz [19]. The maximum input power of the tripler is assumed to be about 4 mW, which is the state-of-art output power at 320 GHz [15].

We have first analyzed a 1-Thz range tripler by using the Schottky varactor UVa 2T2, whose parameters are shown in Table I. The theoretical maximum efficiency is shown in Fig. 5. According to our results the theoretical maximum output power at 960 GHz seems to be about 150 μW with optimum embedding impedances $Z_1 = (25 + j65)\Omega, Z_2 = j20\Omega, Z_3 = (25 + j15)\Omega$. The maximum output power is 4 dB higher than the power obtained by using Kollberg's model [10]. The difference cannot be explained by the increased maximum current due to the edge effects, because the maximum output power of the doubler for 960 GHz is almost the same with both models (about 60 μW with input power of 2 mW at 480 GHz). The difference between these results can be explained only by the fact that the empirical i^6-law works only with doublers and it cannot be employed for frequency multipliers with a higher multiplication factor.

According to our results the optimum bias voltage seems to be about zero (no bias required!) and the efficiency decreases slightly, when the negative bias voltage is applied. This can be understood because the electron velocity saturation limits the modulation of the transition front during the pump cycle. This means that when a large negative bias voltage is applied over the varactor, the voltage during the pump cycle never reaches the maximum capacitance modulation region (see Fig. 6), which is employed in effective multiplication. The maximum possible voltage modulation can be estimated from the maximum modulation of the transition front, which is given by

$$\Delta W = \frac{v_m}{2f_{\text{out}}} \tag{13}$$

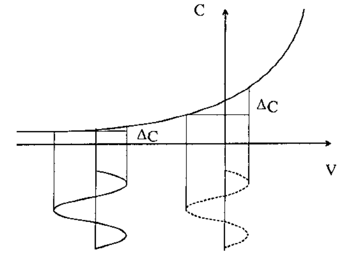

Fig. 6. Schematic of the voltage modulation during a pump cycle, with large negative bias voltage (solid line) and with zero bias (dashed line).

where v_m is the maximum velocity of electrons and f_{out} is the output frequency of the multiplier. The maximum modulation of the transition front for the 1 THz multiplier is about 0.15 μm. However, for UVa 2T2 the thickness of the epitaxial layer is about three times the maximum modulation of the transition front, which means that most of the epitaxial layer is never depleted. The undepleted epitaxial layer works only as a series impedance by decreasing the efficiency of the frequency multiplication [20].

Because the varactor UVa 2T2 is not an optimum choice for the 1-THz-range multiplier, we have analyzed the tripler by using an optimized varactor, whose parameters are given in Table I. The optimized varactor differs from UVa 2T2 only in the thickness of the epitaxial layer (the doping concentration of the epitaxial layer was not optimized). The thickness of the optimized varactor is chosen so that the epitaxial layer is just totally depleted, when the most negative voltage during a pump cycle is reached. For a 1-Thz range tripler with zero bias and with 4 mW input power the maximum negative voltage during a pump cycle is about −2.0 V, which means that the optimum thickness of the epitaxial layer is 0.2 μm. Due to the decreased series resistance of the epitaxial layer the maximum theoretical efficiency of the optimized varactor is about 10%, when the input power is 1.8 mW as shown in Fig. 5. The maximum output power of the optimized varactor is about 250 μW, with optimum embedding impedances $Z_1 = (30 + j55)\Omega, Z_2 = j20\Omega, Z_3 = (20 + j10)\Omega$. The output power of the optimized varactor is 2 dB higher than that generated by UVa 2T2.

IV. CONCLUSION

The equivalent circuit of the millimeter and submillimeter wave Schottky varactor is strongly affected by the edge effects of the small-area circular anode. In this work the electron velocity saturation has been modeled by limiting the velocity of the transition front between the depleted and the undepleted layers. By using this model the maximum conduction current of the varactor is given by the actual area of the transition front, which is always larger than the area of the anode. The equivalent circuit of the Schottky varactor includes also correction factors for the junction capacitance and for the series resistance.

The agreement between the theoretical results and the experimental results is good in the case of the two diode balanced doubler for 160 GHz. The model can be used to analyze any submillimeter wave Schottky varactor frequency multiplier, because the model is independent from the multiplication factor and from the type of the Schottky varactor. According to our simulations the maximum theoretical all-solid-state power at 1 THz seems to be about 250 μW, when an optimized Schottky varactor tripler is employed.

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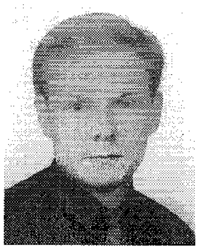

Jyrki T. Louhi received the degree of Diploma Engineer (M.Sc.) with honors and the Licentiate of Technology degree in electrical engineering from the Helsinki University of Technology (HUT), Espoo, Finland, in 1991 and 1994, respectively. He is currently working towards the Ph.D. degree.

Since 1991, he has worked as a research engineer at the Radio Laboratory HUT. His research interest is the development of submillimeter-wave frequency multipliers as well as the modeling of submillimeter-wave Schottky varactors.

Antti V. Räisänen (S'76–M'81–SM'85–F'94), for a photograph and a biography, see p. 954 of this TRANSACTIONS.